Logarithmic Properties

Key Points:

- The inverse of an exponential function is a logarithmic function, and the inverse of a logarithmic function is an exponential function.

- Logarithmic equations can be written in an equivalent exponential form, using the definition of a logarithm and vice versa.

- A ***logarithm*** is the exponent to which b must be raised to get x; written $y = \log_b(x)$.

- The ***logarithmic function*** is $\boldsymbol{y = log_b(x)}$ if and only if $b^y = x$, for $x > 0$, $b > 0$, and $b \neq 1$.

- The common logarithm is $y = \log(x)$ if and only if $10^y = x$, for $x > 0$.

- The natural logarithm is $y = \ln(x)$ if and only if $e^y = x$, for $x > 0$.

- The product rule for logarithms is as follows:

$$\log_b(M * N) = \log_b(M) + \log_b(N).$$

- The quotient rule for logarithms is as follows:

$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$

- The power rule for logarithms is as follows: $\log_b(M^N) = N\log_b(M)$

- The change-of-base formula is $\log_b(M) = \frac{\log_n(M)}{\log_n(b)}$ for $n > 0, n \neq 1$, and $b \neq 1$.

Logarithmic Properties Video

- Converting from Logarithmic Form to Exponential Form : Examples 1-2
- Converting from Exponential Form to Logarithmic Form : Examples 3-5
- Using the Product Rule for Logarithms: Examples 6-7
- Using the Quotient Rule for Logarithms: Examples 8-9
- Using the Power Rule of Logarithms: Examples 10-13
- Expanding Logarithms: Examples 14-16
- Combining Logarithms: Examples 17-19
- Change of Base Formula: Examples 20-21

Practice Exercises

Follow the directions for each exercise below:

1. Rewrite $\boldsymbol{ln(7r * 11st)}$ in expanded form.

2. Rewrite $\boldsymbol{log_8(x) + log_8(5) + log_8(y) + log_8(13)}$ in compact form.

3. Rewrite $\boldsymbol{log_m\left(\frac{67}{83}\right)}$ in expanded form.

4. Rewrite $\boldsymbol{ln(z) - ln(x) - ln(y)}$ in compact form.

5. Rewrite $\ln\left(\frac{1}{x^5}\right)$ as a product.

6. Rewrite $-\mathbf{log}_y\left(\frac{\mathbf{1}}{\mathbf{12}}\right)$ as a single logarithm.

7. Use properties of logs to expand $\log\left(\frac{r^2s^{11}}{t^{14}}\right)$.

8. Use properties of logarithms to expand $\ln\left(2b\sqrt{\frac{b+1}{b-1}}\right)$.

9. Condense the expression $\boldsymbol{5\,ln(b) + ln(c) + \frac{ln(4-a)}{2}}$ to a single logarithm.

10. Condense the expression $\boldsymbol{3\,log_7(v) + 6\,log_7(w) - \frac{log_y(u)}{3}}$ to a single logarithm.

11. Rewrite $\boldsymbol{log_3(12.75)}$ to base e.

Answers:

1. $\ln(7) + \ln(r) + \ln(11) + \ln(s) + \ln(t)$

2. $\log_8(65xy)$

3. $\log_m(67) - \log_m(83)$

4. $\text{Ln}\left(\frac{z}{xy}\right)$

5. $-5\ln(x)$

6. $\log_y(12)$

7. $\log(r^2) + \log(s^{11}) - \log(t^{14})$

8. $\ln(2) + \ln(b) + \frac{\ln(b+1)-\ln(b-1)}{2}$

9. $\ln(c * b^5 * \sqrt{4-a})$

10. $\log_7\left(\frac{v^3 w^6}{\sqrt[3]{u}}\right)$

11. $\log_e(9.75)$